Graphics Pipelines for Boundary-Aware Reduced Simulation and Scratch-Anisotropy Imagery

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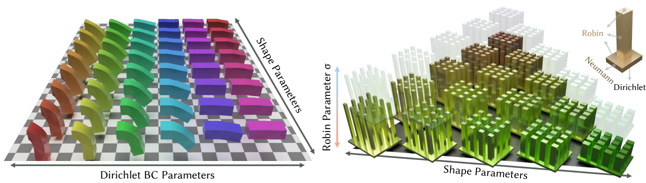
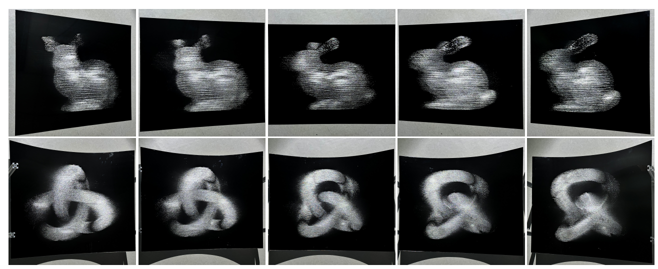


(a) Boundary-aware neural model reduction over shape–boundary spaces.



(b) Scratch-anisotropy reflectors showing view-dependent shaded imagery.

Figure 1: Two graphics pipelines with explicit physical parameters. (a) geometry and Dirichlet, Neumann, or Robin boundary conditions jointly determine neural reduced eigenbases for elastodynamic and heat-transfer PDE simulation. (b) optimized scratch density and orientation on planar or curved aluminum reflectors create view-dependent specular imagery as the camera moves under fixed lighting.

Abstract

We summarize two graphics pipelines: one for boundary-aware reduced simulation and one for scratch-anisotropy design of specular imagery. The first develops neural model reduction for PDE systems whose boundary configurations vary with geometry or design choices. The second translates target view-dependent shading into scratch orientation, density, and manufacturable curves on specular substrates. Together, the two projects show how graphics methods make physically meaningful quantities explicit within concrete computational pipelines. We describe each project's formulation, representative results, and practical scope.

CCS Concepts

• **Computing methodologies** → **Physical simulation**; **Rendering**; • **Applied computing** → **Arts and humanities**.

Keywords

Reduced-order modeling, Implicit neural representation, Boundary conditions, Eigenanalysis, Physical simulation, Computational Arts, Scratch Optimization, Anisotropy Reflection Design

1 Overview

This extended abstract presents two completed graphics systems side by side: *Boundary-aware Neural Model Reduction for PDEs* [Liao et al. 2026] and *S^3 Imagery: Specular Shading from Scratch-Anisotropy* [Shen et al. 2025]. The boundary-aware system extends neural eigenanalysis and reduced-order modeling by incorporating boundary conditions directly into the operator formulation. The scratch-anisotropy system designs scratch-based specular reflectors by coupling anisotropic reflection modeling with fabrication-oriented curve generation.

A shared thread is the explicit treatment of physical quantities in each computational pipeline. In boundary-aware model reduction, Dirichlet, Robin, and mixed boundary configurations are part of the eigenproblem, enabling reduced bases over joint shape–boundary spaces for acoustic, elastic, and thermal simulation. In scratch imagery, target view-dependent shading is converted into scratch orientation, density, view and light configurations, and manufacturable ODE-grown curves on specular substrates. The two pipelines remain separate in problem setting and validation, while both expose the quantities needed to compute their physical behavior.

2 Boundary-aware Neural Model Reduction

Reduced-order modeling accelerates PDE-based simulation by projecting physical fields onto a low-dimensional basis, often built from

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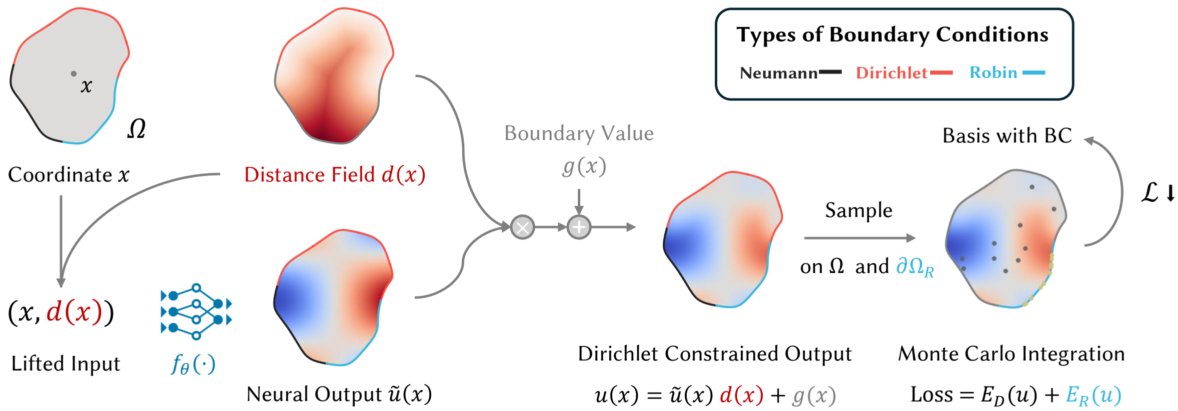

Figure 2: Boundary-aware neural eigenanalysis pipeline.

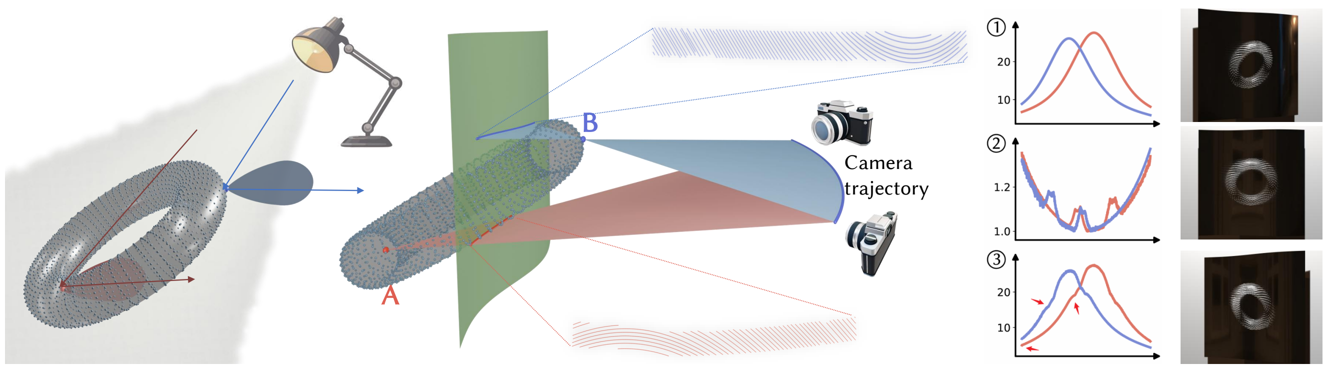

Figure 3: Scratch-anisotropy design pipeline.

eigenmodes of differential operators [Quarteroni and Rozza 2014]. Mesh-free, geometry-agnostic simulation reduces dependence on a fixed discretization [Modi et al. 2024]. Shape Space Spectra evaluates spectra across shape-parameterized domains [Chang et al. 2025], with the standard Rayleigh–Ritz form yielding natural Neumann boundary conditions. Boundary-aware neural model reduction extends this line to Laplace-type eigenanalysis where Dirichlet, Robin, and mixed boundary conditions change the operator, as in clamps, supports, pressure-release openings, and convective boundaries.

As shown in Fig. 2, the central step is to make boundary conditions part of the eigenanalysis. For shape parameter s and boundary parameter β, Dirichlet, Neumann, and Robin regions define a different admissible space and operator $\mathcal{L}(s, \beta)$. Dirichlet conditions are imposed by writing $u(x) = \tilde{u}(x)d(x) + g(x)$, with $d(x) = 0$ on the constrained boundary. Because distance-field gradients can be discontinuous, the neural field is evaluated on the lifted input $\tilde{u}(x) = f_\theta(x, d(x))$.

Robin conditions enter through the variational energy: the surface term $E_R[u] = \int_{\partial\Omega_R} (\frac{1}{2}\sigma u^2 - \psi u)\, dS$ augments the interior Dirichlet energy and yields $\partial_n u + \sigma u = \psi$ on $\partial\Omega_R$. Interior and boundary integrals are estimated by Monte Carlo sampling. Experiments vary boundary parameters in analytic Robin problems, acoustic cavities with sliding pressure-release openings, reduced elastic simulation with fixed supports, and heat-sink diffusion with Robin convection.

3 Specular Shading from Scratch-Anisotropy

S^3 Imagery addresses view-dependent appearance from manufactured scratched reflectors. Scratch or abrasion holography creates virtual 3D impressions by arranging scratches whose highlights move consistently with the viewer [Brand 2011; Regg et al. 2010]. S^3 extends these pipelines with luminance control for shaded virtual objects on planar and developable curved aluminum substrates.

As shown in Fig. 3, scratch design starts from sampled virtual surface points $\{p_i\}$ with target radiance $\mathcal{L}(p_i, V)$. Given a base surface, light position, and camera trajectory $P_{\text{cam}}(t)$, each point is projected onto the reflector to form a centerline $P_i(t)$. Scratch attributes are chosen so the reflected radiance near that centerline matches $\mathcal{L}(p_i, V_i(t))$ over the viewing path.

The core computation separates orientation from density. A scratch gives maximum reflection when its direction $D_i(t)$ is perpendicular to the half vector $H_i(t) = (V_i(t)+L_i(t))/\|V_i(t)+L_i(t)\|$. Using the scratch reflection model of Shen et al. [Shen et al. 2023], S^3 sets density by $\rho_i(t) = \mathcal{L}(p_i, V_i(t))/(S_i(t)|V_i(t) \cdot N_i(t)|)$ under a non-overlap assumption. For fabrication, bundles of short curves follow $\dot{x}_i(t) = D_i(t)$; angle correction, clustered dithering, and birth-and-death updates convert continuous density into manufacturable scratch counts. The validation includes rendered and carved planar and developable reflectors, with limitations tied to excessive density and curved-surface toolpath planning.

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